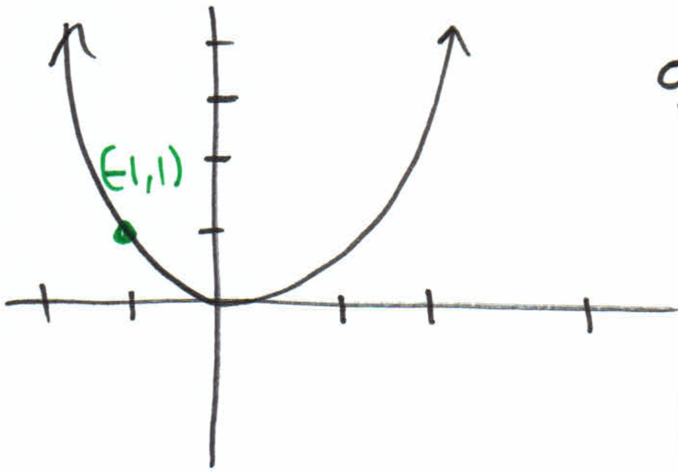
Lecture 11/1/23 : Combining Transformations (1)

#2 : $f(x) = x^2$



What happens to $(-1,1)$ if :

a) Shift up by 1 unit then stretch vertically by a factor of 3 $g(x)$

$(-1,1) \longmapsto (-1,2) \longmapsto (-1,6)$

Stretch vertically by a factor of 3 and shift up by 1. $h(x)$

$(-1,1) \longmapsto (-1,3) \longmapsto (-1,4)$

So we must be careful w/ doing different vertical transformations at the same time.

c) $g(x) = 3(f(x) + 1)$

$h(x) = 3f(x) + 1$

#3 : Similar but w/ horizontal! Do first after lecture! Do #3 w/ them.

#3

a) Answer: $\left(\frac{-2}{3}, 1\right)$

b) Answer: $\left(\frac{-4}{3}, 1\right)$

c) Yes!

d) Shift then compress $\boxed{f(3x+1)}$

compress then shift $\boxed{f(3(x+1)}$

Order of Transformations

It DOES NOT matter if we do horizontal transformations first or vertical transformations first. But it does matter in what order we do particular vertical trans. and vertical trans. In general we will follow this order:

The function $a \cdot f(b(x+h)) + k$ is obtained by

1. a) horizontal stretch / compress / Reflect across y-axis by b
   b) Shift horizontally by h — This is opposite of what we think.
2. c) vertical stretch / compress / reflect across x-axis by a
   d) shift vertically. by k

Ex
Describe the following graphs

a) $y = m(\frac{1}{5}x) - 3$

~~Hor. Comp~~
① Hor. Str. by 5
② Vertical shift down 3

b) $y = 3m(x) + 14$

① Vertical stretch by 3
② Shift up by 14

$\boxed{-m(\frac{1}{4}(x+3)) + 20}$

c) $y = -\left(m\left(\frac{1}{4}(x+3)\right) - 20\right)$

① ~~horizontal shift left by~~ 3 Horizontal St. by 4
② Horizontal shift left by 3
③ reflect across y-axis
④ Shift up by 20.

Some abstract nonsense: let $f(x)$ be a function.

1) Find a formula for $g(x)$ where $g(x)$ is f first compressed, stretched horizontally by a factor of b and then shift horiz. by ~~a~~ $h>0$ units to the left.

Warning it is not $g(x) = f(bx+h)$! As we can see in #3.

Sol: To first compress/stretch horizontally f ~~first~~ we ~~com~~ do $f(bx)$. To shift by h units to the left we need to swap x for $(x+h)$.

$$\boxed{f(b(x+h))}$$

2) Find a formula for shifting by $h>0$ units to the left and then compressing by a factor of b.

Sol: To shift f we write $f(x+h)$. ~~not~~ To compress/stretch horizontally by b we replace x with bx, so

$$\boxed{f(bx+h)}$$